MJC-4 ③

Potential Difference

"The work done by an external agent in carrying a unit +ve test charge from one point to the other point in an electric field is called the potential difference between those points."

The potential difference between the two points A and B is the potential energy per unit charge (W/q).

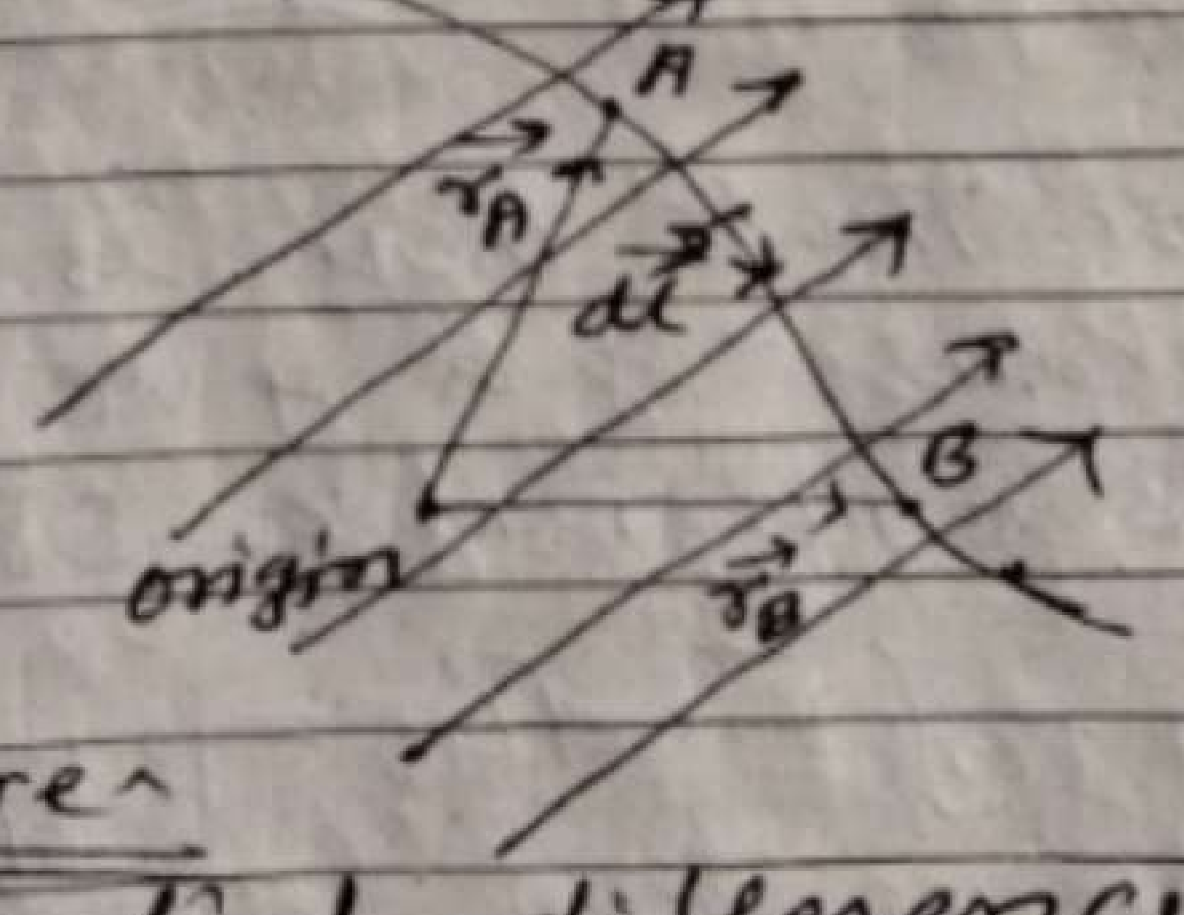
It is denoted by V_{AB}.

$$V_{AB} = \frac{W}{q} = -\int_A^B \vec{E} \cdot \vec{dl} \quad \text{—— (1)}$$

For example:

It is a point charge q at the origin produced a field $\vec{E}$.

i.e. $$E = \frac{q}{4\pi \varepsilon_0 r^2}$$



Figure

Potential difference due on a uniform Electric field

Then the Potential difference between the Points A and B is given as;

$$V_{AB} = -\int_A^B \frac{q}{4\pi\epsilon_0 r^2} dr$$

$$= -\frac{q}{4\pi\epsilon_0}\int_A^B \frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] = (V_B - V_A)$$

Therefore, the Potential difference between the Points A and B can supposed as the Potential (or absolute Potential) of B with respect to the Potential (or absolute Potential) of A. In Point charge case, the reference is taken at Infinity with Zero potential.

Potential Gradient

Suppose that there is an electric field $\vec{E}$ because of a +ve charge placed at the origin of a Sphere. Then

$$V = -\int \vec{E} \cdot d\vec{l} = \frac{q}{4\pi\epsilon_0 r}$$

Figure: illustrate that, as the distance from the charge to the point increase the Potential decreases.

$\int \text{div}\, \vec{E}\, dv = \frac{1}{\varepsilon} \int \rho\, dv \Rightarrow \int \text{div}\, \vec{E}\, dv - \frac{1}{\varepsilon} \int \rho\, dv = 0$

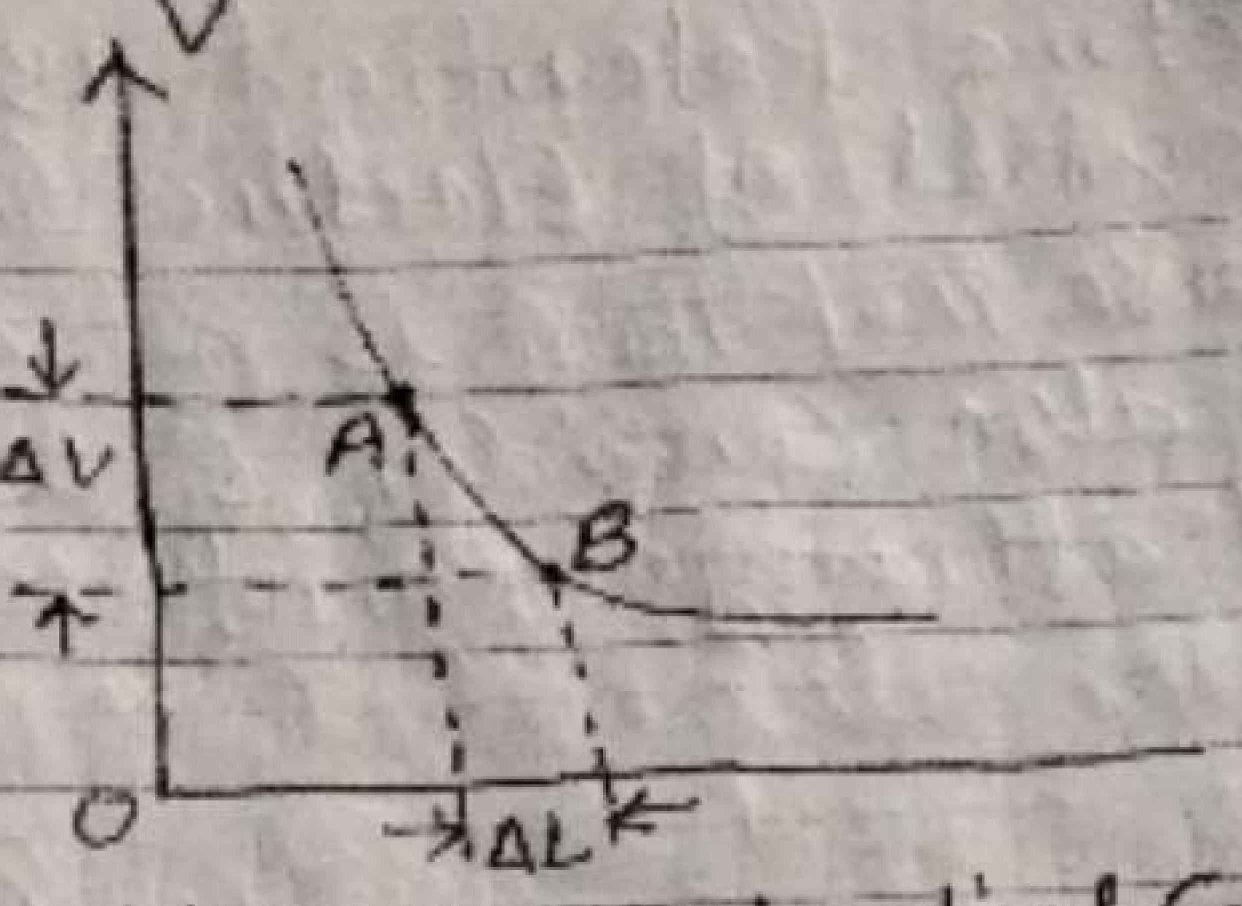


Figure Potential Gradient

The Potential difference between two points is obtained by the line integral of $\vec{E}$ between the two points. It is written for an elementary length ΔL as;

$$\therefore V_{AB} = \Delta V = -\vec{E} \cdot \vec{\Delta L}$$

Therefore, an inverse relation i.e. the change of Potential ΔV, along the elementary length ΔL, must be related to $\vec{E}$, such as $\Delta L \to 0$.

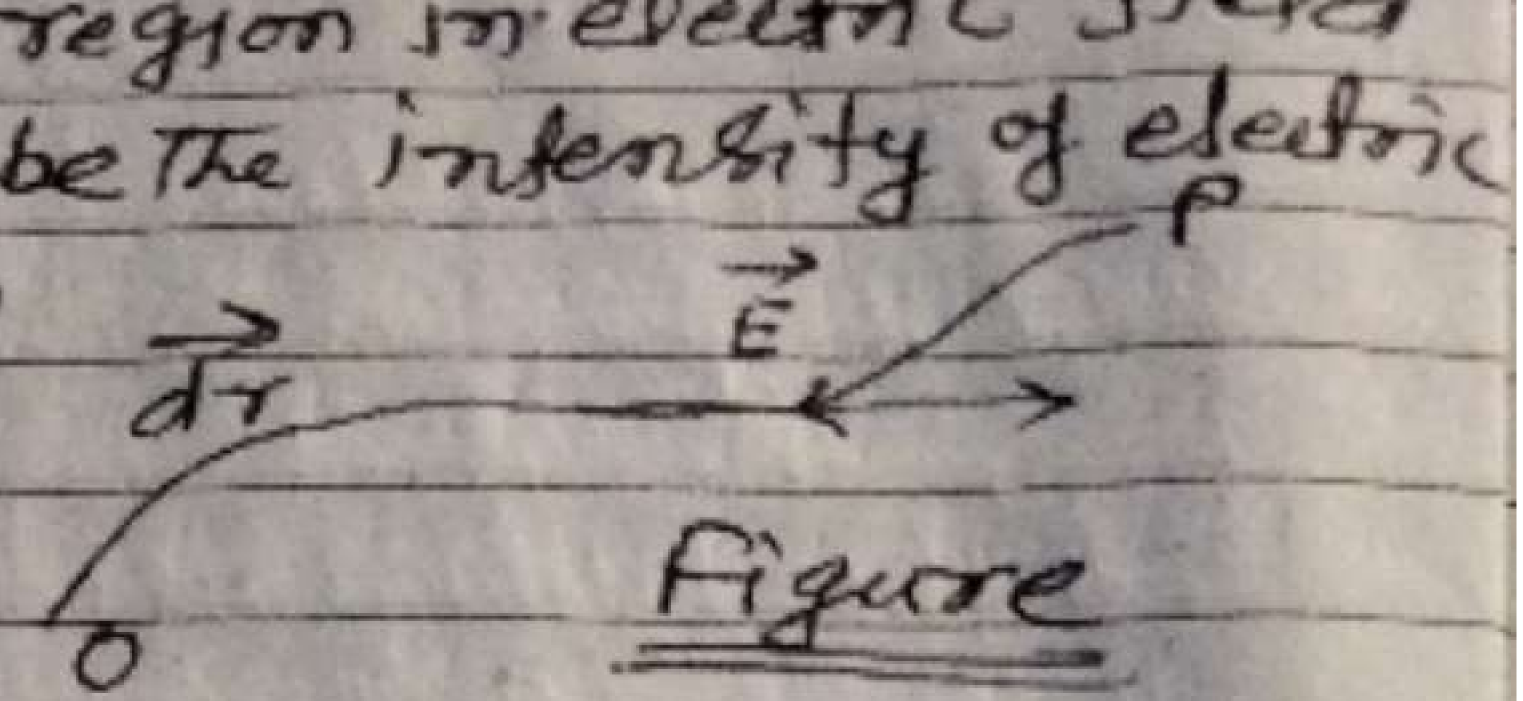
Hence the Potential gradient is defined as the rate of change of Potential with respect to the distance

Thus $\dfrac{dV}{dL} = \lim\limits_{\Delta L \to 0} \dfrac{\Delta V}{\Delta L}$ = Potential Gradient

In the other words, the Potential gradient is also defined as the slope of the graph of Potential against distance at a point where elementary length is considered.

Electric Potential as line Integral of Electric field

Suppose that there is a region in electric field and also consider $\vec{E}$ be the intensity of electric field at any point.



Figure

When a +ve test charge q_0 is moved from point P to point Q in the opposite direction of the electric field

Then the external force on test charge $\vec{F} = -q_0\vec{E}$.

Hence, the work is displacing the test-charge through a small displacement $d\vec{r}$ becomes:

$$dW = \vec{F}\cdot\vec{dr} = -q_0\vec{E}\cdot\vec{dr} \quad \text{--- (1)}$$

The total work done in displacing the charge from point P to Q is given by:

$$W_{PQ} = -q_0\int_P^Q \vec{E}\cdot d\vec{r} \quad \text{--- (2)}$$

Where the integral extends along the path from P to Q. Thus, the potential difference between two points P to Q becomes:

$$V_Q - V_P = \frac{W_{PQ}}{q_0} = -\int_P^Q \vec{E}\cdot\vec{dr} \quad \text{--- (3)}$$

When a point P is taken at infinity, the reference level for zero potential which means $V_P = 0$, then the potential at point Q can be expressed as;

$$V_Q = -\int_\infty^Q \vec{E}\cdot\vec{dr} \quad \text{--- (4)}$$

Hence the electric potential is expressed as the -ve of the line integral of the electric field from infinity to a specified point at any point in an electric field.