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Paper No. : Physics at Nanoscale - III
Module : **Calculation of Density of States (DOS)**

Description of Module	
Subject Name	Physics
Paper Name	Physics at Nanoscale - III
Module Name/Title	Quantum Mechanics and Nanoscale Structures
Module Id	

MODULE III

3.3 Calculation of Density of states (DOS)

- 3.3.1 DOS in 3D
- 3.3.2 DOS in 2D
- 3.3.3 DOS in 1D
- 3.3.4 DOS in 0D

CALCULATION OF DENSITY OF STATES (DOS): Quantum Wells, Wires and Dots

3.3.1 DOS for 3 dimensions (Bulk)

Consider the volume in "k" space

$$V = \frac{4}{3}\pi k^3 \tag{3.44}$$

where for a particle in this space

$$k_x = \frac{2\pi}{L_x} \quad , \quad k_y = \frac{2\pi}{L_y} \text{ and } k_z = \frac{2\pi}{L_z} \tag{3.45}$$

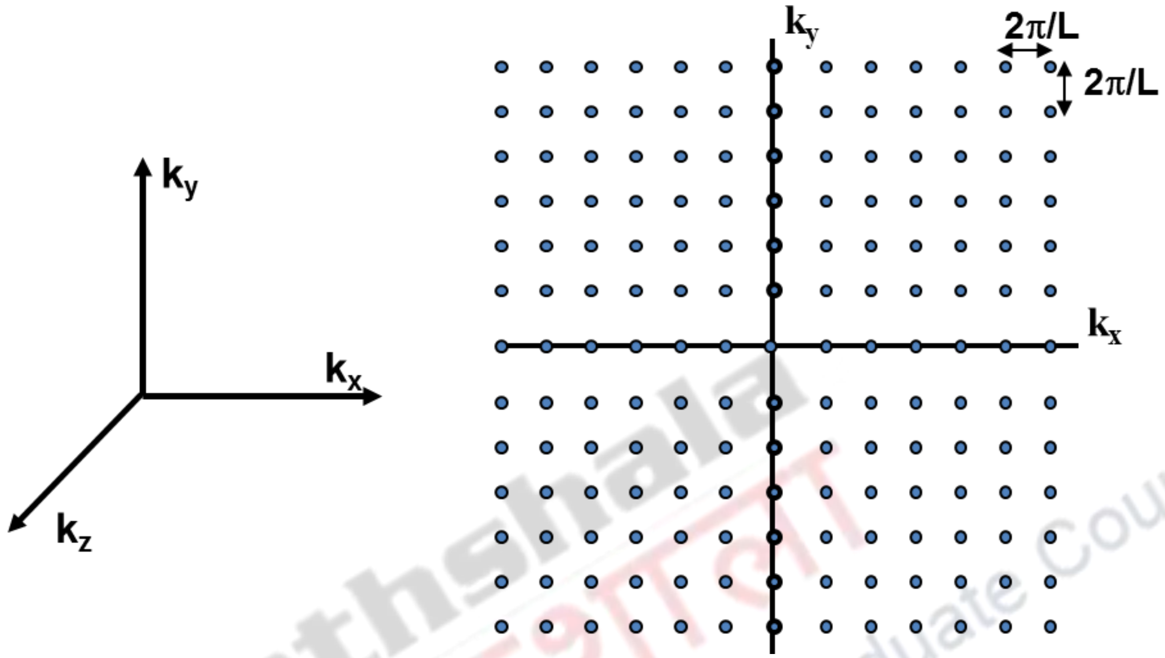


Figure . Electron state is defined by a point in k-space.

Note that the 2π arises from the constraints of periodic boundary conditions as proposed to the more general $n\pi$ where n=0, 1, 2, 3... The volume of a given mode is then= $k_x k_y k_z$. The number of modes (N) in the sphere is,

$$N = \frac{V}{k_x k_y k_z} = \frac{\frac{4}{3}\pi k^3}{8\pi^3} L_x L_y L_z \qquad (3.46)$$

Say the particle in an electron and we consider spin (up and down), then we multiply N by 2.

$$N' = 2N = 2\frac{\frac{4}{3}\pi k^3}{8\pi^3} L_x L_y L_z$$

$$= \frac{k^3}{3\pi^2} L_x L_y L_z \qquad (3.47)$$

is the total number of states in sphere. Now consider the density

$$\rho = \frac{N'}{L_x L_y L_z}$$

$$= \frac{k^3}{3\pi^2}$$

is the number of states per unit volume. Now consider the energy defined as

$$\rho' = \frac{d\rho}{dE} = \frac{d\left(\frac{k^3}{3\pi^2}\right)}{dE} = \frac{1}{3\pi^2}\frac{dk^3}{dE}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

Therefore, $$\rho' = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E} \qquad (3.48)$$

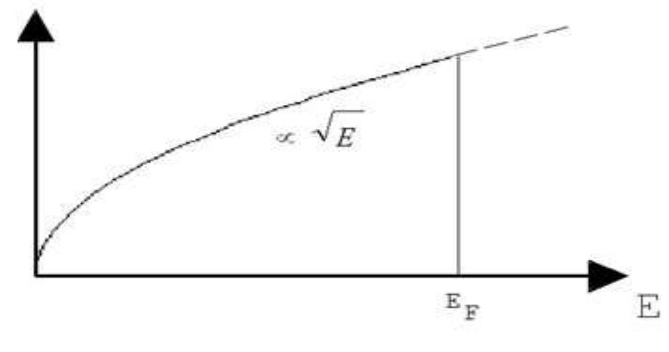


Figure . Density of states in 3 dimension (Eq.3.48)

3.3.2 DOS in Two Dimensions (well)

Here we have 1D that is quantized. Let's us assume it is the z-direction. The total energy of this system is a sum of the energy along the quantized direction plus the energy along the other 2 free directions. It is expressed as

$$E_T = \frac{\hbar^2 k_z^2}{2m} + \frac{\hbar^2 k^2}{2m} = E_n + E_{x,y} \qquad (3.49)$$

where$k^2 = k_x^2 + k_y^2$ and $k_z = \frac{n\pi}{L_z}$. Consider now an area in k-space

$$A = \pi k^2$$

where for the particle

$$k_x = \frac{2\pi}{L_x}$$

$$k_y = \frac{2\pi}{L_y}$$

The area of a given mode is then $k_x k_y$ with the total number of modes (N) in the area being

$$N = \frac{\pi k^2}{4\pi^2} L_x L_y = \frac{k^2 L_x L_y}{4\pi}$$

Again if the particle is an electron and we consider spin, multiply by 2 to get

$$N' = 2N = \frac{k^2 L_x L_y}{2\pi} \qquad (3.50)$$

Consider now density, $\rho = \frac{N'}{L_x L_y} = \frac{k^2}{2\pi}$, and we have $k = \sqrt{\frac{2mE}{\hbar^2}}$

And the energy density is defined as

$$\rho' = \frac{d\rho}{dE} = \frac{m}{\pi\hbar^2}$$

This is the energy density of the sub-band for a given k_z or (E_n). For each successive k_z there will be an additional $\frac{m}{\pi\hbar^2}$ and hence another subband. Therefore the density of the states is written

$$\rho'_{2D=} \frac{m}{\pi\hbar^2} \sum_n \Theta(E - E_n) \qquad (3.51)$$

Where Θ is the heavy side function.

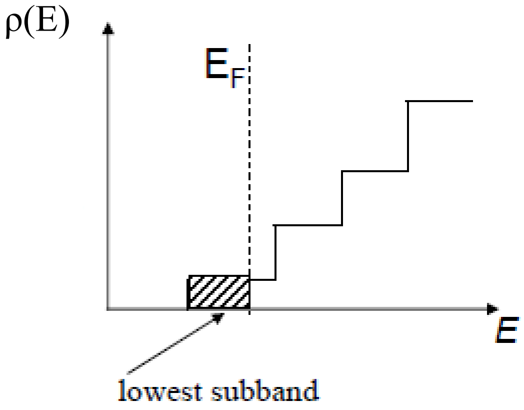


Figure Density of states in 2 dimension. Shaded area presents occupied states.

3.3.3 DOS in One Dimensions (Wire)

Consider now the situation where there are two dimensions confined and only 1 degree of freedom (say the x-direction). The total energy of the system can be written as

$$E_T = \frac{\hbar^2 k_z^2}{2m} + \frac{\hbar^2 k_z^2}{2m} + \frac{\hbar^2 k^2}{2m} = E_n + E_m + E_x \qquad (3.52)$$

where

$$k = k_x = \frac{2\pi}{L_x}$$

Furthermore along the confined directions,

$k_z = \frac{n\pi}{L_z}$, and $k_y = \frac{m\pi}{L_y}$ and m, n are integers. Consider a length *2k*. The number of modes along this length is

$$N = \frac{2k}{k_x} = \frac{2k}{\left(\frac{2\pi}{L_x}\right)} = \frac{kL_x}{\pi} \qquad (3.53)$$

Number of states along the line

Now if we consider an electron again, ensure to take into account the spin degeneracy

$$N' = 2N = \frac{2kL_x}{\pi}$$

Now a density is

$\rho = \frac{N'}{L_x} = \frac{2k}{\pi} = \frac{2}{\pi}\sqrt{\frac{2mE}{\hbar^2}}$ is the number of states per unit length.

Consider the energy density

$$\rho' = \frac{d\rho}{dE} = \frac{1}{\pi}\sqrt{\frac{2m}{E\hbar^2}}$$

This is the energy density for a given n, m value, the expression taking into account all m, n combination is

$$\rho'_{1D}\,\frac{1}{\pi}\sqrt{\frac{2m}{\hbar^2}}\sum_{n,m}\frac{1}{\sqrt{E-E_{n,m}}}\Theta\left(E-E_{n,m}\right) \qquad (3.54)$$

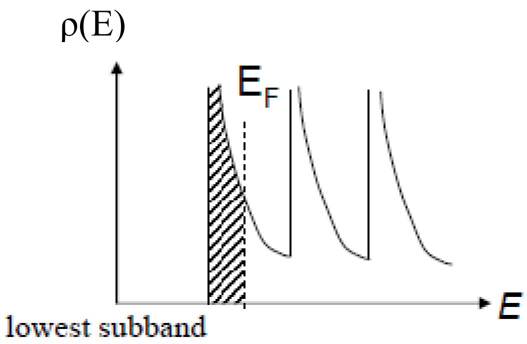


Figure Density of states in 1 dimension. Shaded area presents occupied states.

3.3.4 Zero dimensions (Quantum Dot)

Here since all three dimensions are confined. The density of states is basically a series of delta functions. The total energy of the system is

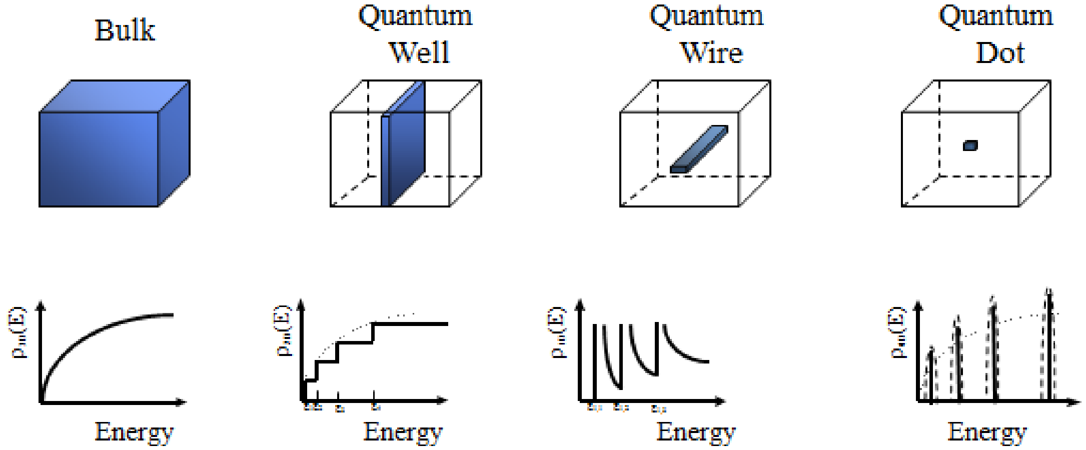
$$E_T = \frac{\hbar^2 k_x^2}{2m} + \frac{\hbar^2 k_y^2}{2m} + \frac{\hbar^2 k_z^2}{2m} = E_m + E_n + E_o \qquad (3.55)$$

where m, n, o are integers and

$$k_x = \frac{m\pi}{L_x}, k_y = \frac{m\pi}{L_y}, k_z = \frac{m\pi}{L_z}$$

The density of states is

$$\rho'_{0D} = \delta\left(E - E_{m,n,o}\right) \qquad (3.56)$$



3.3.5 More density of states

Density of states in the conduction band

For this we need to know the probability that an electron will occupy a given stats of energy E. The Probability, P(E), is referred as the Fermi Dirac distribution. In addition we need to know the density of states (ρ'). The density of states has units of number of unit volume per unit energy. Therefore $\rho' dE$ is the number of states per unit volume. The number of occupied states at a given energy per unit volume is therefore

$$n_e(E) = P_e(E)\rho'(E)dE \qquad (3.57)$$

where the Fermi Dirac distribution is

$$P_e(E) = \frac{1}{1+e^{\frac{E-E_F}{kT}}} \qquad (3.58)$$

Here E_F is the Fermi energy.

the total concentration of electrons in the conduction band is therefore the integral over all available energies

$$n_T = \int_{E_c}^{\infty} P_e(E)\rho'(E)dE \qquad (3.59)$$

whereE_c is the energy where conduction band starts. For the case of three dimensional material

$$\rho' = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E} \qquad (3.60)$$

Taking account into conduction band begins, the density of states can be written as

$$\rho' = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E - E_c} \qquad (3.61)$$

the total concentration of electrons in the conduction band is given as $n_T =$

$$\frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{\frac{3}{2}}\int_{E_c}^{\infty}\frac{1}{1+e^{\frac{E-E_F}{kT}}}\sqrt{E - E_c}dE \qquad (3.62)$$

The integral is called the Fermi integral or Fermi Dirac integral.

Consider the case where, $E - E_F \gg KT$ and the Fermi Dirac distribution function becomes

$$P_e(E) = \frac{1}{1 + e^{\frac{E-E_F}{kT}}} = \frac{1}{e^{\frac{E-E_F}{kT}}} = e^{-\frac{E-E_F}{kT}}$$

which is the Boltzmann distribution law. Therefore

$$n_T = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{\frac{3}{2}}\int_{E_c}^{\infty} e^{-\frac{E-E_F}{kT}}\sqrt{E - E_c}dE \qquad (3.63)$$

Now change variables and let $x = \frac{E-E_c}{kT}$, such that $E = E_c + xkT$ and $dE = kTdx$. Note also that the limits of integration will change accordingly. This leads to

$$n_T = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{\frac{3}{2}}\int_0^{\infty} e^{-\frac{[(E-E_C)+(E_C-E_F)]}{kT}}\sqrt{xkT}dx$$

$$n_T = \frac{1}{2\pi^2}\left(\frac{2m_e kT}{\hbar^2}\right)^{\frac{3}{2}}\int_0^{\infty} e^{-x}\sqrt{x}dx$$

$\int_{E_c}^{\infty} e^{-x}\sqrt{x}dx$is gamma function equal to $\Gamma\left(\frac{3}{2}\right)$, therefore

$$n_c = n_T = \frac{1}{2\pi^2}\left(\frac{2m_e kT}{\hbar^2}\right)^{\frac{3}{2}} e^{-\frac{(E_C-E_F)}{kT}}\Gamma\left(\frac{3}{2}\right)$$

$$N_C = \frac{1}{2\pi^2}\left(\frac{2m_e kT}{\hbar^2}\right)^{\frac{3}{2}}\Gamma\left(\frac{3}{2}\right) \qquad (3.64)$$

$$n_c = n_T = N_C e^{-\frac{(E_C-E_F)}{kT}} \qquad (3.65)$$

This is the expression for the effective density of states of the conduction band.

Density of states in the valance band

The number of holes at a given energy per unit volume is given as

$$n_h(E) = P_h(E)\rho'(E)dE \tag{3.66}$$

where we use the relation

$$P_e(E) + P_h(E) = 1$$

$$P_h(E) = 1 - \frac{1}{1 + e^{\frac{E-E_F}{kT}}}$$

and the density of states is

$$\rho' = \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E_V - E} \tag{3.67}$$

Where E$_v$ is the energy where valance band starts. The total concentration of holes in the valance band is the integral over all energies.

$$p_T = \int_{-\infty}^{E_v} P_h(E)\rho'(E)dE$$

$$p_T = \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{\frac{3}{2}}\int_{-\infty}^{E_v}\left(1 - \frac{1}{1+e^{\frac{E-E_F}{kT}}}\right)\sqrt{E_V - E}dE \tag{3.68}$$

Since generally speaking E<E$_F$

$$p_T = \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{\frac{3}{2}}\int_{-\infty}^{E_v}\left(1 - \frac{1}{1+e^{-\frac{E_F-E}{kT}}}\right)\sqrt{E_V - E}dE$$

$$p_T = \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{\frac{3}{2}}\int_{-\infty}^{E_v} e^{-\frac{E_F-E}{kT}}\sqrt{E_V - E}dE \tag{3.69}$$

As with the conduction band case earlier, make a change of variables by letting, $x = \frac{E_v - E}{kT}$ as well as $E = E_v - kTx$ and $dE = -kTdx$. Note that the limits of integration change accordingly resulting in

$$p_T = \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{\frac{3}{2}}\int_{\infty}^{0} e^{-\frac{[(E_F-E_v)+(E_v-E)]}{kT}}\sqrt{kTx}(-kT)dx$$

$$p_T = \frac{1}{2\pi^2}\left(\frac{2m_hkT}{\hbar^2}\right)^{\frac{3}{2}} e^{-\frac{(E_F-E)}{kT}}\int_{\infty}^{0} e^{-x}\sqrt{x}dx$$

$$p_T = \frac{1}{2\pi^2}\left(\frac{2m_hkT}{\hbar^2}\right)^{\frac{3}{2}} e^{-\frac{(E_F-E)}{kT}}\Gamma\left(\frac{3}{2}\right) \tag{3.70}$$

$$p_T = N_v e^{-\frac{(E_F-E)}{kT}}$$

This is the effective density of the states in the valance band.

Summary

Fermi level of an intrinsic semiconductor

If the bulk semiconductor is intrinsic, there has been no doping of the material and hence no extra electrons or holes anywhere. in this situation

$$n_c = p_v$$

therefore

$$N_C e^{-\frac{(E_C-E_F)}{kT}} = N_v e^{-\frac{(E_F-E)}{kT}}$$

$$e^{-\frac{(-E_C+E_F+E_F-E_v)}{kT}} = \left(\frac{m_h}{m_e}\right)^{\frac{3}{2}}$$

$$E_F = \frac{E_C+E_v}{2} + \frac{3}{4}kTln\left(\frac{m_h}{m_e}\right) \qquad (3.71)$$

One can therefore see that at T=0 the Fermi energy of an intrinsic semiconductor is at the halfway point between the top of the valance band and the bottom of the conduction band.

Density of states in the conduction band

We start with the Fermi Dirac distribution for electrons and also the density of states

$$\rho'_{2D}=\frac{m_e}{\pi\hbar^2}\sum_n \Theta(E-E_n) \qquad (3.72)$$

Consider only one of the subband. In this case the density of states simplifies to

$$\rho'_{2D}=\frac{m_e}{\pi\hbar^2} \qquad (3.73)$$

Now recall from the previous section that the number of states at a given energy per unit volume

$$n_e(E) = P_e(E)\rho'(E)dE$$

the total concentration of electrons in this first subband is the integral over all available energies. Rather than use ntot as before let's just stick to n_c from the start

$$n_c = \int_{E_c}^{\infty} P_e(E)\rho'(E)dEdE \qquad (3.74)$$

Where $P_e(E) = \frac{1}{1+e^{\frac{E-E_F}{kT}}}$ is the Fermi Dirac distribution. Therefore we have

$$n_c = \frac{m_e}{\pi\hbar^2}\int_{E_c}^{\infty}\frac{1}{1+e^{\frac{E-E_F}{kT}}}dE \tag{3.75}$$

Since the band really begins at en as opposed to *Ec* like in the bulk the integral change from

$$n_c = \frac{m_e}{\pi\hbar^2}\int_{E_n}^{\infty}\frac{1}{1+e^{\frac{E-E_F}{kT}}}dE$$

if now E-E$_F$>>kT

$$n_c = \frac{m_e}{\pi\hbar^2}\int_{E_n}^{\infty}e^{-\frac{E-E_F}{kT}}dE$$

$$n_c = -\frac{m_e kT}{\pi\hbar^2}e^{-\frac{E-E_F}{kT}}\Big|_{E_n}^{\infty} \tag{3.76}$$

this leads the final expression to the carrier density of the nth subband i.e.

$$n_c = \frac{m_e kT}{\pi\hbar^2}e^{-\frac{E_n-E_F}{kT}} \tag{3.77}$$

Density of states in the valance band

As with the conduction band case we need the probability of occupying a given state in the valance band. This denoted $P_h(E)$ and is evaluated from

$$P_e(E) + P_h(E) = 1$$

Therefore

$$P_h(E) = 1 - \frac{1}{1+e^{\frac{E-E_F}{kT}}}$$

The number of states at a given energy per unit volume

$$n_h(E) = P_h(E)\rho'(E)dE$$

If, as in the conduction band case, we consider only the nth subband $\rho' = \frac{m_h}{\pi\hbar^2}$ and for the simplicity the first. The total concentration of holes in this first subband is the integral over all energies. we get

$$p_V = \int_{-\infty}^{E_V}P_h(E)\rho'(E)dE$$

$$= \frac{m_h}{\pi\hbar^2}\int_{-\infty}^{E_V}\left(1-\frac{1}{1+e^{\frac{E-E_F}{kT}}}\right)dE \tag{3.78}$$

Since the n^th subband begins at E_n rather than E_V, therefore

$$p_V = \frac{m_h}{\pi\hbar^2}\int_{-\infty}^{E_n}\left(1-\frac{1}{1+e^{\frac{E-E_F}{kT}}}\right)dE \qquad (3.79)$$

and since for the valance band $E < E_F$

$$p_V = \frac{m_h}{\pi\hbar^2}\int_{-\infty}^{E_n}\left(1-\frac{1}{1+e^{-\frac{E_F-E}{kT}}}\right)dE \qquad (3.80)$$

$$p_V = \frac{m_h}{\pi\hbar^2}\int_{-\infty}^{E_n} e^{-\frac{E_F-E}{kT}}dE$$

$$p_V = \frac{m_{hkT}}{\pi\hbar^2}e^{-\frac{E_F-E_n}{kT}} \qquad (3.81)$$

Fermi level position :2D

The procedure for finding the Fermi level position is the same as in the 3D

Consider a spherical volume of

$$V_k = \frac{4}{3}\pi k^3$$

The volume of a given mode was, recall $k_x k_y k_z$ where

$$k_x = \frac{2\pi}{L_x}$$

$$k_y = \frac{2\pi}{L_y}$$

$$k_z = \frac{2\pi}{L_z}$$

The number of modes or states in the given sphere is then

$$N = \frac{V_k}{k_x k_y k_z} = \frac{\frac{4}{3}\pi k^3}{8\pi^3}L_x L_y L_z$$

For an electron, multiply this by 2 to account for spin we have

$$N' = 2N = \frac{k^3}{3\pi^2}L_x L_y L_z$$

Now the density is

$$\rho = \frac{N'}{L_x L_y L_z} = \frac{k^3}{3\pi^2}$$

is the number of states per unit volume and the energy density is given as

$$\rho' = \frac{d\rho}{dE} = \frac{1}{2\pi^2}\left(\frac{2m}{ħ^2}\right)^{\frac{3}{2}}\sqrt{E}$$

$$\rho_2' = \frac{d\rho}{dk} = \frac{k^2}{\pi^2}$$

Divide by 2 to go back to only 1 spin orientation since in an optical transition spin slips are generally forbidden

$$\rho_1' = \frac{k^2}{2\pi^2}$$

The expression applies to either conduction band or valance band. Applying the following equivalence

$$\rho_j(E)dE = \rho_1'(k)dk$$

$$\rho_j(E) = \rho_2'(k)\frac{dk}{dE}$$

$$= \frac{k^2}{2\pi^2}\frac{dk}{dE}$$

whereρ_j is the desired joint density of the states. Now from the conservation of momentum, transition in k are vertical such that the initial k value in the valance band is the same k value as in the conduction band (k_a=k_b=k) where ka is the k value in the valence band and kb is the value in the conduction band. The energy of the initial state in the valance band is

$$E_a = E_v - \frac{ħ^2 k_a^2}{2m_h}$$

Likewise the energy of the final state in the conduction band is

$$E_b = E_c + \frac{ħ^2 k_b^2}{2m_e}$$

The energy of the transition is

$$E = E_b - E_a$$

$$E = \left(E_c + \frac{ħ^2 k_b^2}{2m_e}\right) - \left(E_v - \frac{ħ^2 k_a^2}{2m_h}\right)$$

$$E = (E_c - E_v) + \frac{\hbar^2 k_b^2}{2m_e} + \frac{\hbar^2 k_a^2}{2m_h}$$

$$E = E_g + \frac{\hbar^2 k^2}{2}\left(\frac{1}{m_e} + \frac{1}{m_h}\right)$$

$$\frac{dE}{dk} = \hbar^2 k \frac{m_e + m_h}{m_e m_h}$$

leading to the desired expression

$$\frac{dk}{dE} = \frac{1}{\hbar^2 k}\left(\frac{m_e m_h}{m_e + m_h}\right)$$

Since

$$\rho_j(E) = \frac{k^2}{2\pi^2}\left(\frac{dk}{dE}\right)$$

$$\rho_j(E) = \frac{k^2}{2\pi^2}\frac{1}{\hbar^2 k}\left(\frac{m_e m_h}{m_e + m_h}\right)$$

$$\rho_j(E) = \frac{k\mu}{2\pi^2\hbar^2}$$

Where for notational simplicity we have used the reduced mass

$$\mu = \frac{m_e m_h}{m_e + m_h}$$

Therefore

$$E = E_g + \frac{\hbar^2 k^2}{2\mu}$$

$$k = \frac{\sqrt{2\mu(E - E_g)}}{\hbar}$$

$$\rho_j(E) = \frac{\mu k}{2\pi^2\hbar^2}$$

$$= \frac{\mu}{2\pi^2\hbar^3}\sqrt{2\mu(E - E_g)}$$

$$= \frac{1}{4\pi^2}\left(\frac{2\mu}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{(E-E_g)}$$

2D Well

Area in k-space

$$A_k = 4\pi k$$

Where the area occupied by a given mode or state is $k_y k_z$ .Here we assume that k_x represents the confined direction

$$k_y = \frac{2\pi}{L_y}$$

$$k_z = \frac{2\pi}{L_z}$$

Together, the number of modes in the area is

$$N = \frac{A_k}{k_y k_z} = \frac{4\pi k^2}{4\pi^2} L_y L_z = \frac{k^2}{\pi} L_y L_z$$

Multiply by 2 to account for spin

$$N' = 2N = \frac{2k^2}{\pi} L_y L_z$$

Now consider the density

$$\rho = \frac{N'}{L_y L_z} = \frac{2k^2}{\pi}$$

With the energy density given by

$$\rho' = \frac{d\rho}{dE} = \frac{m}{\pi\hbar^2}$$

$$\frac{d\rho}{dk} = \frac{4k}{\pi}$$

Starting with the energy density

$$\rho' = \frac{d\rho}{dk} = \frac{4k}{\pi}$$

Divide by 2 to get rid of the spin since formally speaking, spin flip optical transitions are forbidden

$$\rho' = \frac{\rho_2'}{2} = \frac{2k}{\pi}$$

Now applying the following equivalence

$$\rho_j(E)dE = \rho_2'(k)dk$$

One obtains

$$\rho_j(E) = \rho_1'(k)\frac{dk}{dE}$$

$$\rho_j(E) = \frac{2k}{\pi}\frac{dk}{dE}$$

where$\rho_j(E)$ is the desired joint density of states. As before in the 3D case, the conservation of momentum means that transition in k-space are vertical. That is the initial k value in the valance band is the same as the final k value in the conduction band ($k_a = k_b = k$) where $k_a(k_b)$ is the valance (conduction) band values.

The energy of the initial state in the valance band is

$$E_a = E_{n2} - \frac{\hbar^2 k^2}{2m_h}$$

Likewise the energy of the final state in the conduction band is

$$E_b = E_{n1} - \frac{\hbar^2 k^2}{2m_e}$$

The transition energy is

$$E = E_b - E_a$$

$$E = E_{n1} - E_{n2} + \frac{\hbar^2 k^2}{2}\left(\frac{1}{m_e} + \frac{1}{m_h}\right)$$

$$E = E_g + \frac{\hbar^2 k^2}{2\mu}$$

This leads to

$$\frac{dE}{dk} = \frac{\hbar^2 k}{\mu}$$

Or

$$\frac{dk}{dE} = \frac{\mu}{\hbar^2 k}$$

Such that when replaced into our main expression the desired expression for the joint density of states is

$$\rho_j(E) = \frac{2\mu}{\hbar^2 \pi}$$

1D wire

Consider the length in k-space

$$L_k = 2k$$

The length occupied by a given mode or state is k_x where

$$k_x = \frac{2\pi}{L_x}$$

The number of states in the given length is

$$N = \frac{L_k}{k_x} = \frac{2k}{2\pi} L_x = \frac{kL_x}{\pi}$$

Multiply this by 2 to account for spin, we get total number of states as

$$N' = 2\frac{kL_x}{2\pi} = \frac{kL_x}{\pi}$$

Consider the density ie number of states per unit length

$$\rho = \frac{N'}{L_x} = \frac{2k}{\pi}$$

And the energy density is given by

$$\rho' = \frac{d\rho}{dE} = \frac{1}{\pi}\sqrt{\frac{2m}{\hbar^2}} E^{-\frac{1}{2}}$$

Or alternately

$$\frac{d\rho}{dk} = \frac{2}{\pi}$$

Starting with the energy density

$$\rho_2' = \frac{d\rho}{dk} = \frac{2}{\pi}$$

Divide by 2 to consider only one spin orientation since spin flip transition are generally forbidden

$$\rho_1' = \frac{\rho_2'}{2} = \frac{1}{\pi}$$

Now apply the following equivalence

$$\rho_j(E) = \rho_1'(k)\frac{dk}{dE}$$

$$\rho_j(E) = \frac{1}{\pi}\frac{dk}{dE}$$

where$\rho_j(E)$ is the desired joint density of states. As before in the 3D and 2D case, the conservation of momentum means that transition in k-space are vertical so that $k_a = k_b = k$) where $k_a(k_b)$ is the valance (conduction) band values.

The energy of the initial state in the valance band is

$$E_a = E_{m2,n2} - \frac{\hbar^2 k^2}{2m_h}$$

Likewise the energy of the final state in the conduction band is

$$E_b = E_{m1,n1} - \frac{\hbar^2 k^2}{2m_e}$$

The transition energy is

$$E = E_b - E_a$$

$$E = E_{m1,n1} - E_{m2,n2} + \frac{\hbar^2 k^2}{2}\left(\frac{1}{m_e} + \frac{1}{m_h}\right)$$

$$E = E_g + \frac{\hbar^2 k^2}{2\mu}$$

This leads to

$$\frac{dE}{dk} = \frac{\hbar^2 k}{\mu}$$

Or

$$\frac{dk}{dE} = \frac{\mu}{\hbar^2 k}$$

Such that when replaced into our main expression the desired expression for the joint density of states is

$$\rho_j(E) = \frac{\mu}{\hbar^2 \pi} \frac{1}{k}$$

Now to continue towards our final expression we express k fully. Since

$$E = E_g + \frac{\hbar^2 k^2}{2\mu}$$

$$\frac{1}{k} = \frac{\hbar}{\sqrt{2\mu(E - E_g)}}$$

This leads to the final expression for the joint density of states

$$\rho_j(E) = \frac{1}{\pi} \sqrt{\frac{\mu}{2\hbar^2}} \frac{1}{\sqrt{(E - E_g)}}$$