MIC-4

Electric field equal to Gradient of Potential :

The electric Potential is a Scalar function for The description of The electrostatic field. It is equal to The work done by The electric field in moving a Small charge from an arbitrary Point A in The field To a 'reference' Point R. Per unit charge. For The description of The electrostatic field The electric Potential is a Scalar function.

$$V_A = \int_A^R E \cdot dl \ (V) \qquad \text{—— (1)}$$

on adding equⁿ (1) with The law of Conservation of energy.

$$\oint_c E \cdot dl = 0 \qquad \text{—— (2)}$$

Potential can be expressed as

$$V_{(r)} = \frac{q}{4\pi\varepsilon_0 r} \text{ (Reference point at infinity) } V \qquad \text{—— (3)}$$

The Volume, Surface or Potential of a given distribution of line charges at a Point P of The field is given as :

$$V_P = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, dv}{r} \ ; \quad V_P = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\, ds}{r}$$

$$V_P = \frac{1}{4\pi\varepsilon_0}\int_l \frac{q'\, dl}{r} \ V \qquad \text{—— (4)}$$

In the electrostatic field, the potential difference or voltage is defined as:

$$V_{AB} = V_A - V_B = \int_A^B E \cdot dl \; V \quad \text{—— (5)}$$

from equ^n (5) the electric field strength is obtained from the electric scalar potential as:

$E = -\text{grad } V = -\nabla . V$

In rectangular from

$$E = -\text{grad } V = -\nabla . V$$

$$= -\left(\frac{\partial v}{\partial x} v_x + \frac{\partial v}{\partial y} v_y + \frac{\partial v}{\partial z} v_z\right) \; V/m$$

Example- If the potential $V = \frac{10}{r^2} \sin\theta \cdot \cos\phi$ Then find out the electric field intensity and flux density at $(4, \pi/2, 0)$.

Sol-

The electric field is specified as,

$$\overline{E} = -\nabla V = \left[\frac{\partial v}{\partial r} \hat{a}_r + \frac{1}{r}\frac{\partial v}{\partial \theta}\hat{a}_\theta + \frac{1}{r}\frac{\partial v}{\partial \phi}\hat{a}_\phi\right]$$

$$= \frac{20}{r^3}\sin\theta \cdot \cos\phi\, \hat{a}_r - \frac{10}{r^3}\cos\theta\cos\phi\, \hat{a}_\theta +$$

$$\frac{10}{r^3}\sin\theta\sin\phi\, \hat{a}_\phi$$

The field intensity at $(4, \pi/2, 0)$ is given by:

$$\overline{E} = \frac{20}{4^3}\sin\left(\frac{\pi}{2}\right)\cos 0\,\hat{a}_r -$$

$$\frac{10}{4^3}\cos\left(\frac{\pi}{2}\right)\cos 0\,\hat{a}_\theta + \frac{10}{4^3}\sin\left(\frac{\pi}{2}\right)\sin 0\,\hat{a}_\phi = \frac{5}{16}\hat{a}_r$$

Thus, the flux density is given as

$$\overline{D} = \varepsilon_0 \overline{E} = 8.854 \times 10^{-12} \times \frac{5}{16}\hat{a}_r$$

$$= 2.77\,\hat{a}_r \; \underline{PC/m^2}$$

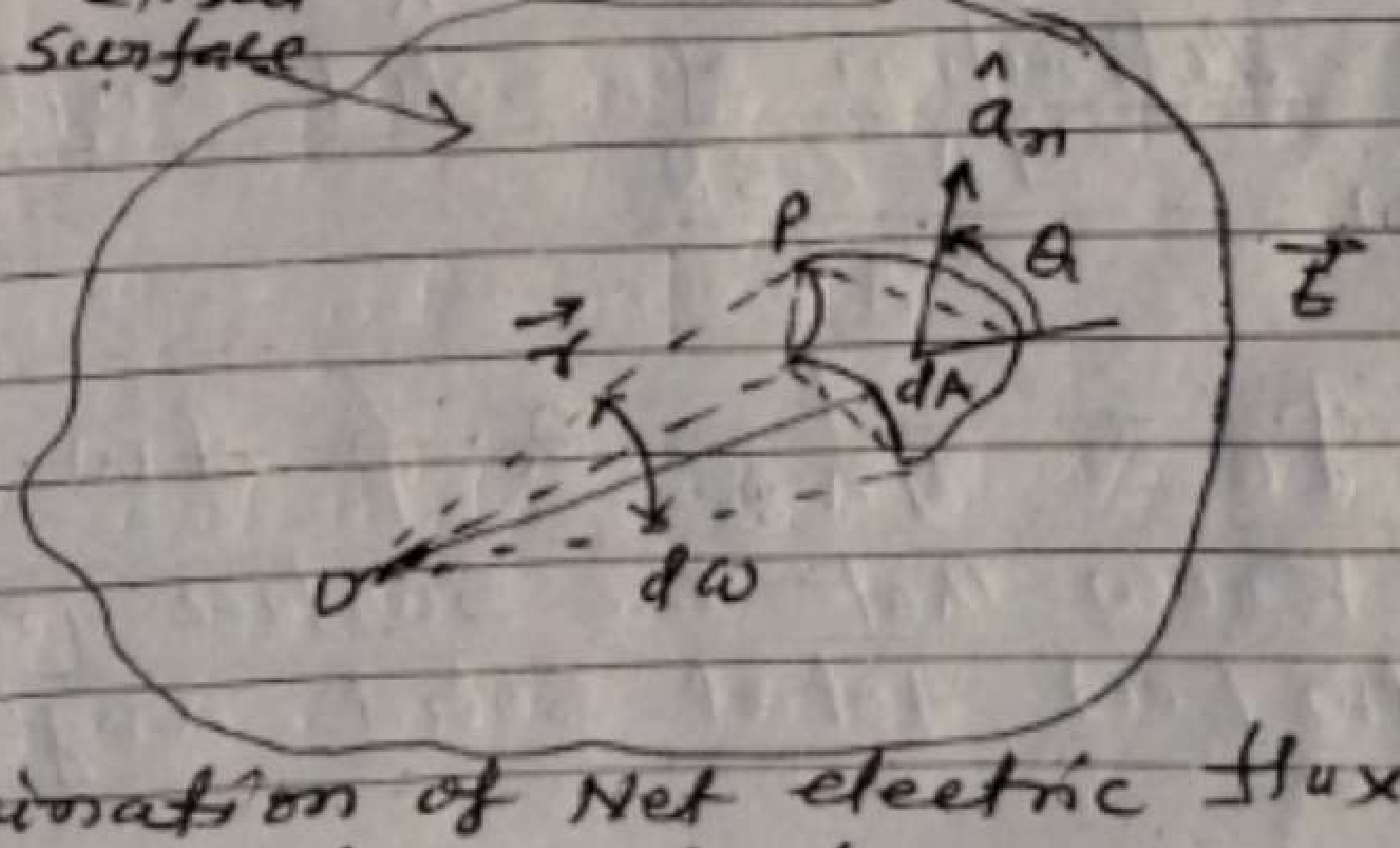
Gauss' Law

According to Gauss' law. "The total electric displacement or electric flux Through any closed Surface surrounding charges is equal to The net +ve charge enclosed by That Surface".
It is also known as Gauss' flux Theorem.

In free Space The Surface integral of The electric field intensity over any closed hypothetical surface (i.e. Gaussian Surface) is equivalent to $1/\varepsilon_0$ times The net charge enclosed within The Surface. Hence ε_0 denotes The absolute permitivity of free Space. The total flux Passing Through a closed Surface is Proportional to The charge enclosed within That Surface.

$$\phi_E = \oint_{\text{Surface}} \bar{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Proof: closed surface



Determination of Net electric flux Through a closed Surface.

Suppose There is a charge at Point O. A Point P is located away from 'O' by distance 'r'.
Let There is a close surface of any shape which is passing Through from Point 'P'. This surface is known as Gaussian surface.
According to Coulomb's law, The electric field at a Point is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

Suppose $d\vec{A}$ (area Vector) is small area element at Point 'P'.
By definition of electric flux, we have

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \oint |\vec{E}| \cdot |d\vec{A}| \cos\theta .$$

where θ is angle between $\vec{E}$ and $d\vec{A}$

$$\phi_E = \oint \left(\frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \right) dA \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot Q \cdot \oint \frac{dA \cos\theta}{r^2}$$

By definition of solid angle, we have

$$dw = \frac{dA \cos\theta}{r^2}$$

$$\therefore \phi_E = \frac{1}{4\pi\varepsilon_0} Q \cdot \oint dw$$

we know that the surface integration of dw (or) total solid angle by close surface at inside the close surface is 4π

Thus $\oint d\omega = 4\pi$

$\therefore \quad \Phi_E = \frac{1}{4\pi\varepsilon_0} Q \cdot 4\pi$

$\Phi_E = \frac{Q}{\varepsilon_0}$; i.e. $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$

Hence Gauss's Theorem is Proved.
This equation is the integral form of gauss Theorem.