MJC-7
OPTICS

B.SC (Sem IV)

principle of Superposition of two waves

Let us consider two simple harmonic collinear vibrations vibrations given by

$$y_1 = a_1 \sin\omega t \quad \text{———} \quad (1)$$

and $$y_2 = a_2 \sin(\omega t + \delta) \quad \text{———} \quad (2)$$

Where y_1 y_2 are displacements of a particle at time t under the action of two disturbance having a_1 and a_2 respective amplitudes

ωt and $(\omega t + \delta)$ are phase angles

δ = Phase difference between two vibrations.

ω = frequency

According to the principle of superposition y_1 and y_2 acts independently and their resultant is the algebric sum of the two i.e.

$$y = y_1 + y_2 = a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$$

$$= a_1 \sin\omega t + a_2 \sin\omega t \cos\delta + a_2 \cos\omega t \sin\delta$$

$$= (a_1 + a_2 \cos\delta) \sin\omega t + a_2 \sin\delta \cos\omega t \quad \text{———} \quad (3)$$

Let $a_1 + a_2 \cos\delta = A\cos\phi$ and $a_2 \sin\delta = A\sin\phi$

From eqn. (3),

$$y = A\sin(\omega t + \phi) \quad \text{———} \quad (4)$$

Where A and ϕ are the constants depending on a_1, a_2 and δ

Eqn(4) shows that the resultant displacement has same frequency

squaring and adding component relations of (4), we get.

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\delta}$$

Again dividing the components,

$$\frac{A\sin\phi}{A\cos\phi} = \frac{a_1 \sin\delta}{a_1 + a_2\cos\delta}$$

$$\therefore \tan\phi = \frac{a_1 \sin\delta}{a_1 + a_2\cos\delta} \quad \text{——— (6)}$$

Equation (5) and (6) gives the values of the amplitude and the phase of the resultant vibrations.

Since intensity is proportional to the square of the amplitude,

Therefore, $I = A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

For Intensity to be max; $\cos\delta = +1$ then

$\delta = 0, 2\pi, 4\pi \ldots = 2n\pi$ with $n = 0, 1, 2, 3, \ldots$

$$\boxed{I_{max}} = a_1^2 + a_2^2 + 2a_1a_2 = (a_1+a_2)^2 \quad \text{——— (7)}$$

Max^m Amplitude $A_{max} = a_1 + a_2$

For Minimum Intensity $\delta = -1$, then

$\delta = \pi, 3\pi, 5\pi, \ldots = (2n+1)\pi$

where $n = 0, 1, 2, 3 \ldots$

And. $\boxed{I_{MIN}} = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2$

In terms of path difference

For Max^m. Intensity $\Delta p = 2n\frac{\lambda}{2}$ and

Min Intensity $\Delta p = (2n+1)\lambda/2$

Special Case

when $a_1 = a_2$, $\boxed{I_{max} = 4a^2 \text{ and } A_{max} = 2a}$

$I_{Min} = 0$ and $A_{min} = 0$

$$\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} \text{ and } \frac{A_{Max}}{A_{min}} = \frac{a_1+a_2}{a_1-a_2} = \frac{2a}{0} = \infty$$

$$= \frac{4a^2}{0} = \infty \text{ and}$$

$$\boxed{\frac{I_{max}}{I_{min}} = \infty \text{ and } \frac{A_{max}}{A_{min}} = \infty}$$

Continued ……