VKSU **Example**

'If two Sinusoidal waves given as, $y_1 = 0.03 \cos (7t - 10x)$ m and $y_2 = 0.03 \cos (5t - 8x)$ m were superimposed. Find the group velocity.

**Sol:** Given That

$y_1 = 0.03 \cos (7t - 10x)$ m ——— ①

$y_2 = 0.03 \cos (5t - 8x)$ m ——— ②

Comparing with the wave equⁿ $y = a \cos(\omega t - kx)$

we have,

Amplitude $a = 0.03$ m

Angular frequency, $\omega_1 = 7$; $\omega_2 = 5$

Propagation constant $K_1 = 10$; $K_2 = 8$

∴ Group velocity $V_g = \dfrac{\Delta\omega}{\Delta k} = \dfrac{\omega_1 - \omega_2}{K_1 - K_2}$

$= \dfrac{7-5}{10-8} = 1 \ ms^{-1}$

**Ex** Find the group velocity when the two waves $y_1 = 10 \sin (2\pi t - 5x)$ and $y_2 = 15 \sin (5\pi t + 5x)$ are superimposed $y_1$ and $y_2$ are in m.

**Sol** Given That

$y_1 = 10 \sin (2\pi t - 5x)$ ——— ①

$y_2 = 15 \sin (5\pi t - 5x)$ ——— ②

Comparing with the wave equation $y = a \sin (\omega t - kx)$ we have

Angular frequency $\omega_1 = 2\pi$, $\omega_2 = 5\pi$

Propagation Number (constant)

$K_1 = +5$; $K_2 = -5$

$\therefore$ Group velocity $V_g = \dfrac{\Delta \omega}{\Delta K} = \dfrac{\omega_1 - \omega_2}{K_1 - K_2}$

$$= \dfrac{2\pi - 5\pi}{+5 - (-5)} = \dfrac{-3\pi}{10}$$

$$= -0.3\pi \; m/s$$

**Ex** Let us Consider That There is a Simple harmonic Wave travelling along x-axis is given by $y = 5 \sin 2\pi (0.2t - 0.5x)$

Then find the value of amplitude, frequency, wavelength, Wave velocity, particle velocity, velocity amplitude, Particle acceleration and acceleration amplitude

(x is in metres t in seconds)

**Sol** Given $y = 5 \sin 2\pi (0.2t - 0.5x)$ —— ①

on comparing with above equation

$y = a \sin \dfrac{2\pi}{\lambda} (vt - x)$, we obtain

$y = a \sin 2\pi \left( \dfrac{vt}{\lambda} - \dfrac{x}{\lambda} \right) = a \sin 2\pi \left( nt - \dfrac{x}{\lambda} \right)$

Now Amplitude $\quad a = 5m$

frequency $\quad\quad n = 0.2 \; s^{-1}$

So, $\quad \dfrac{x}{\lambda} = 0.5x$

$\therefore$ wavelength $\lambda = \dfrac{1}{0.5} = 2m$

On putting the value of $n$ and $\lambda$, we obtain

$$n\lambda = 0.2 \times 2 = 0.4 \ m/s$$

Particle velocity $= \dfrac{dy}{dt} = 5\cos 2\pi (0.2t - 0.5x) \times 0.2$

$$= \cos 2\pi (0.2t - 0.5x) \ ms^{-1}$$

Particle velocity amplitude $= 1 \ ms^{-1}$

Particle acceleration $= \dfrac{d^2y}{dt^2} = -\sin 2\pi (0.2t - 0.5x) 0.2$

$$= -0.2 \sin 2\pi (0.2t - 0.5x) \ ms^{-1}$$

Particle acceleration amplitude $= -0.2 \ ms^{-1}$

Ex- There are two sinusoidal waves
$$Y_1 = 0.03 \cos (7t - 10x) \ m \quad \text{and}$$
$$Y_2 = 0.03 \cos (5t - 8x) \ m$$
were superimposed find the group velocity.

Sol=

On comparing above equation with the general equⁿ of wave equation $Y = a\cos(\omega t - Kx)$
we have.

Amplitude $a = 0.03 \ m$

Angular frequency $\omega_1 = 7 ; \quad \omega_2 = 5$

Propagation Constant $K_1 = 10 ; \quad K_2 = 8$

$\therefore$ Group velocity $V_g = \dfrac{\Delta \omega}{\Delta K} = \dfrac{\omega_1 - \omega_2}{K_1 - K_2}$

$$= \dfrac{7-5}{10-8} = 1 \ ms^{-1}$$

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**Ex.** Find out the Phase velocity and group velocity. If the velocity of sea waves is $\sqrt{\left(\dfrac{g\lambda}{2\pi}\right)}$. Also find the group velocity of these waves when the wavelength is 500 meter.

**Sol.** we have

$$V_p = \sqrt{\dfrac{g\lambda}{2\pi}} = \sqrt{\dfrac{g}{k}} \qquad \left(\because k = \dfrac{2\pi}{\lambda}\right)$$

or $\quad \dfrac{\omega}{k} = \sqrt{\dfrac{g}{k}}$

or $\quad \omega = \sqrt{gk}$

$$\dfrac{d\omega}{dk} = \sqrt{g}\,\dfrac{1}{2}\,k^{-1/2} = \dfrac{1}{2}\sqrt{\dfrac{g}{k}}$$

Group velocity,

$$\dfrac{d\omega}{dk} = \dfrac{1}{2}\sqrt{\dfrac{g}{k}} = \dfrac{1}{2}\sqrt{\dfrac{g\lambda}{2\pi}}$$

or $\quad V_g = \dfrac{1}{2}V_p$ ;

$$V_p = \sqrt{\dfrac{g\lambda}{2\pi}} = \sqrt{\dfrac{9.8 \times 500}{2 \times 3.14}}$$

$$= 27.93 \; m/sec.$$

$$V_g = \dfrac{V_p}{2} = \dfrac{27.93}{2} = 13.96 \; m/sec$$