(c) Expression for energy eigen value for simple harmonic oscillator :

The wavefunction, $\psi = e^{-y/2} H(y)$, will have a finite value if $H(y)$ terminates at some value on n. For this to happen, A_{n+2} must vanish for some value of n, i.e.,

$$A_{n+2} = 0$$

or, $$\frac{(2n+1-\lambda)}{(n+1)(n+2)} A_n = 0$$

or, $$2n + 1 - \lambda = 0$$

or, $$\lambda = 2n + 1 \qquad \ldots(22)$$

Putting $\lambda = \frac{2E}{\hbar\omega}$ from equation (8) in equation (22), we get

$$\frac{2E}{\hbar\omega} = 2n + 1$$

or, $$E_n = \left(\frac{n+1}{2}\right)\hbar\omega \qquad \text{... (23)}$$

Putting $\hbar = \frac{h}{2\pi}$ and $\omega = 2\pi\nu$ in equation (23),

We get $$E_n = \left(n + \frac{1}{2}\right)h\nu \qquad \text{... (24)}$$

where $n = 0, 1, 2, 3$ ..... , ω is the angular frequency and ν is the frequency of the classical harmonic oscillator, given by

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Equation (24) is the quantum mechanical energy of simple harmonic oscillator in its n^{th} vibrational state, n being vibrational quantum number.